HAZARD REPORT

WP4 Risk assessment and analysis
4.1 Risk assessment methods & models; 4.2 Improved use of risk assessment methods

MODELLING, IDENTIFICATION AND PREDICTION OF OIL SPILL DOMAINS AT PORT AND SEA WATER AREAS

PRESENTATION AT HAZARD WORKSHOP ORGANIZED BY PSRA ON 15.02.2019 IN GDYNIA

Project Partner: Polish Safety and Reliability Association (PL)

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1. INTRODUCTION

One of the important duties in port activities and shipping is the prevention of oil release from port installations and ships and the spread of oil spills that often have dangerous consequences for port and sea water areas.

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More details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.

2. MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

We denote by $A(t)$ the process of changing hydro-meteorological conditions at the sea water areas where the oil spill happened and distinguish m its states from the set $A = \{1,2,...,m\}$ in which it may stay at the moment t, $t \in <0,T>$, where $T > 0$.

Further, we assume a semi-Markov model of the process $A(t)$ and denote by θ_{ij} its conditional sojourn time in the state i while its next transition will be done to the state j, where $i, j = 1,2,...,m$, $i \neq j$.

2. MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters:

- the vector $[p(0)]$ of probabilities of its initial states at the moment $t = 0$;
- the matrix $[p_{ij}]$ of probabilities of its transitions between the particular states;
- the matrix $[W_{ij}(t)]$ of distribution functions of its conditional sojourn times θ_{ij} at the particular states;

where $i,j = 1,2,...,m$.

2. MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

- the expected values (mean values) M_{ij} of its conditional sojourn times θ_{ij} at the particular states;
- the variances V_{ij} of its conditional sojourn times θ_{ij} at the particular states;

where $i,j = 1,2,...,m$.

2. MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

Having the above parameters of the process of changing hydro-meteorological conditions $A(t)$, $t \in <0,T>$, $T > 0$, this process following characteristics can be determined:

- the distribution functions $W_i(t)$ of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i;
- the mean values M_i of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i;

2. MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

– the variances V_i of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the states i;

– the limit values p_i of the process of changing hydro-meteorological conditions transient probabilities at the particular operation states;

– the total sojourn times $\hat{\theta}_i$ of the process of changing hydro-meteorological conditions at the particular operation states i during the fixed time θ, that have approximately normal distributions;

where $i = 1,2,...,m$.

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

The approach and the results concerned with the survivor search domain at the sea restricted areas determination considered in [Blokus, Kołowrocki 2003] can be modified, developed and applied to oil spill drift trend determination.

First, for each fixed state k, $k = 1,2,\dots,m$, of the process $A(t)$ and time $t \in <0,T>$, where T is time we are going to model the behaviour of the oil spill domain $\bar{D}^k(t)$, we define the central point of this oil spill domain as a point $(x^k(t), y^k(t))$, $t \in <0,T>$, $k = 1,2,\dots,m$, on the plane Oxy that is the centre of the smallest circle, with the radius $r^k(t)$, $t \in <0,T>$, $k = 1,2,\dots,m$, covering this domain.

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

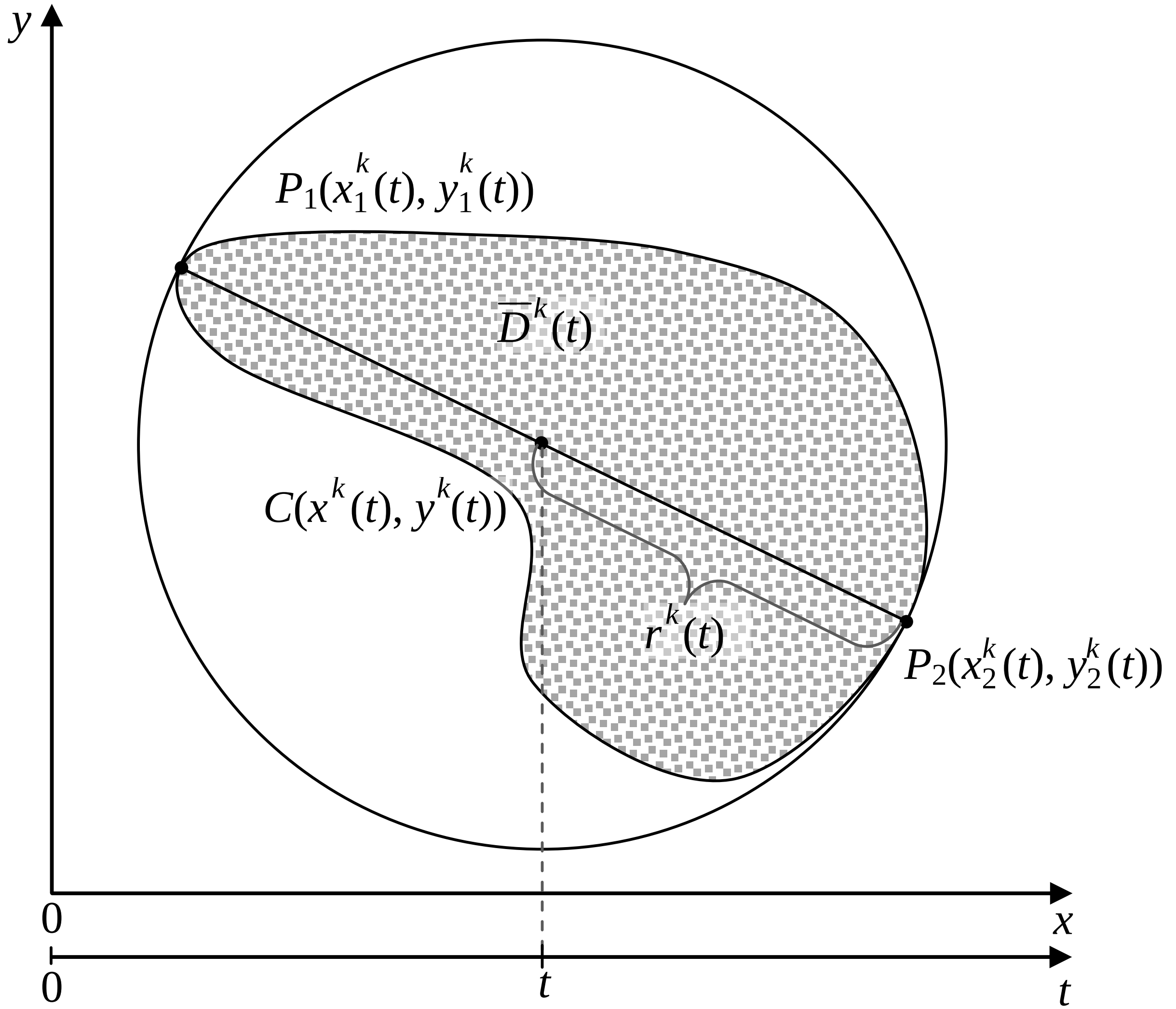

Figure. Interpretation of central point of oil spill definition

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

Further, for each fixed state k, $k = 1,2,\ldots,m$, of the process $A(t)$ and time t, $t \in \langle 0,T \rangle$, we define a two-dimensional stochastic process

$(X^k(t), Y^k(t))$, such that

$$(X^k, Y^k) : \langle 0,T \rangle \to R^2,$$

where $X^k(t)$, $Y^k(t)$ respectively are an abscissa and an ordinate of the plane Oxy point, in which the oil spill central point is placed at the moment t while the process $A(t)$ is at the state k.

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

We set deterministically the central point of oil spill domain in the area in which an accident has happened and an oil release was placed in the water as the origin $O(0,0)$ of the co-ordinate system Oxy.

The value of a parameter t at the moment of accident we assume equal to 0. It means that the process $(X^k(t), Y^k(t))$, is a random two-dimensional co-ordinate (a random position) of the oil spill central point after the time t from the accident moment and that at the accident moment $t = 0$ the oil spill central point is at the point $O(0,0)$, i.e.

$$(X^k(0), Y^k(0)) = (0,0).$$

After some time the central point of the oil spill starts its drift along a curve called a drift curve.

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

In further analysis, we assume that processes

$$(X^k(t), Y^k(t)),\ t \in \langle 0, T \rangle,\ k = 1, 2, \ldots, m,$$

are two-dimensional normal processes

$$N(m_X^k(t), m_Y^k(t), \rho_{XY}^k(t), \sigma_X^k(t), \sigma_Y^k(t)),$$

with varying in time expected values

$$m_X^k(t) = E[X^k(t)],\ \ m_Y^k(t) = E[Y^k(t)],$$

standard deviations $\sigma_X^k(t),\ \ \sigma_Y^k(t)$,

and correlation coefficients $\rho_{XY}^k(t)$,

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

i.e. with the joint density functions

$$\varphi_t^{\,k}(x,y) = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}}$$

$$\exp\{-\frac{1}{2(1-(\rho_{XY}{}^k(t))^2)}[\frac{(x-m_X{}^k(t))^2}{(\sigma_X{}^k(t))^2}$$

$$-2\rho_{XY}{}^k(t)\frac{(x-m_X{}^k(t))(y-m_Y{}^k(t))}{\sigma_X{}^k(t)\sigma_Y{}^k(t)}+\frac{(y-m_Y{}^k(t))^2}{(\sigma_Y{}^k(t))^2}]\},$$

$(x,y)\in R^2,\ t\in <0,T>,\ k=1,2,\ldots,m.$

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

Thus, the points

$$(m_X^k(t), m_Y^k(t)),\ t \in <0,T>,\ k = 1,2,\ldots,m,$$

create a curve K^k called an oil spill central point drift trend which may be described in the parametric form

$$K^k : \begin{cases} x^k = x^k(t) \\ y^k = y^k(t),\ t \in <0,T>. \end{cases}$$

3. MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

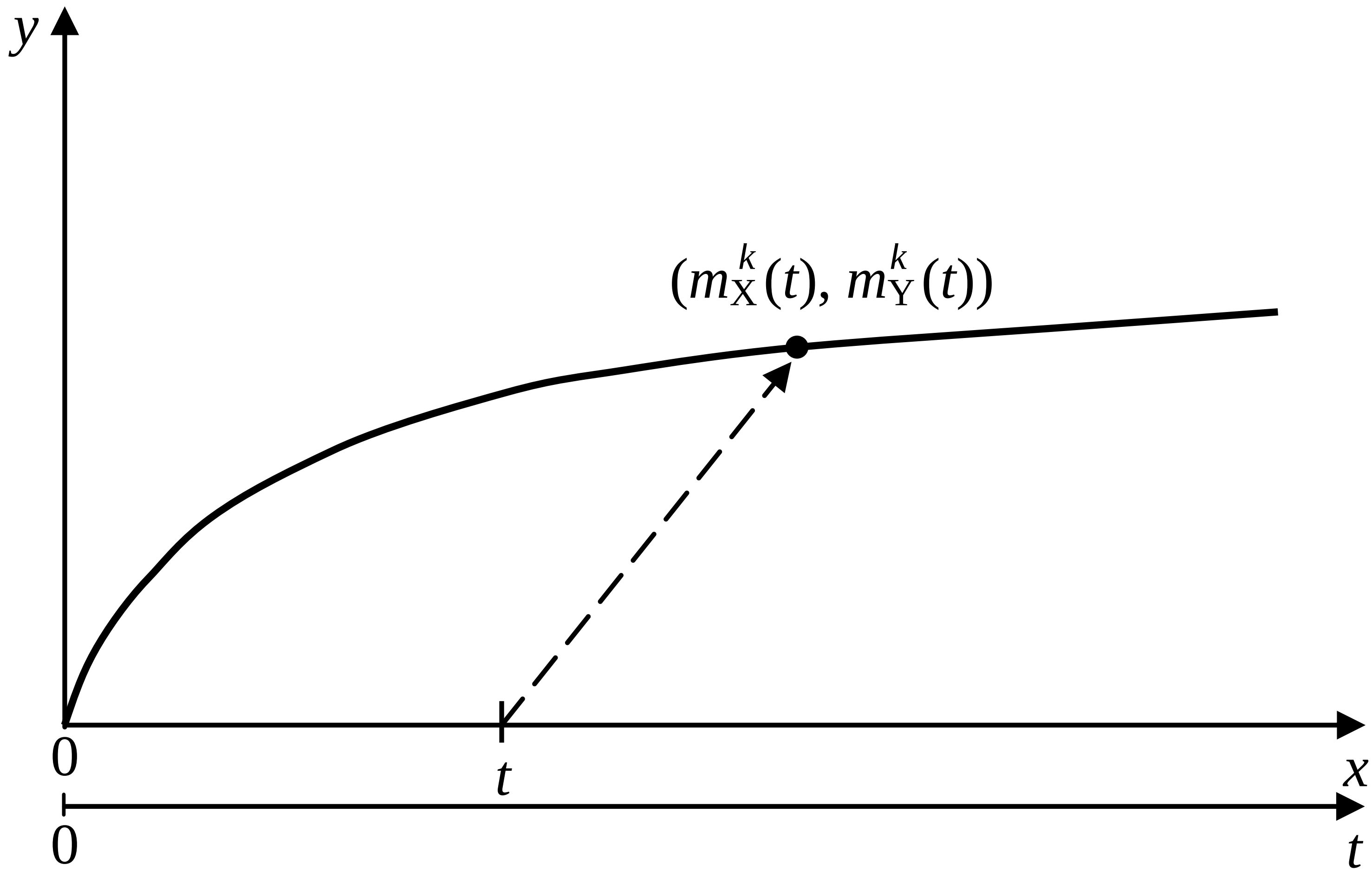

Figure. Oil spill central point drift trend

4. MODELLING OIL SPILL DOMAIN

4.1 PROBABILISTIC APPROACH

We are interested in finding the search domain $D^k(t)$, $t \in <0,T>$, $k = 1,2,\ldots,m$, such that the central point of oil spill domain is placed in it with a fixed probability p. More exactly, we are looking for c such that

$$P((X^k(t),Y^k(t)) \in D^k(t)) = \iint_{D^k(t)} \varphi_t^k(x,y)\,dx\,dy = p,$$

where

$$D^k(t) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2} - 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}] \le c^2\}$$

4. MODELLING OIL SPILL DOMAIN

4.1 PROBABILISTIC APPROACH

is the domain bounded by an ellipse being the projection on the plane $0xy$ of the curve rising as the result of intersection of the density function surface

$$\pi_1^k = \{(x, y, z) : z = \varphi_t^k(x, y), (x, y) \in R^2\}$$

and the plane

$$\pi_2^k = \{(x, y, z) :$$

$$z = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}} \exp[-\frac{1}{2}c^2], \ (x,y) \in R^2\}.$$

4. MODELLING OIL SPILL DOMAIN
4.1 PROBABILISTIC APPROACH

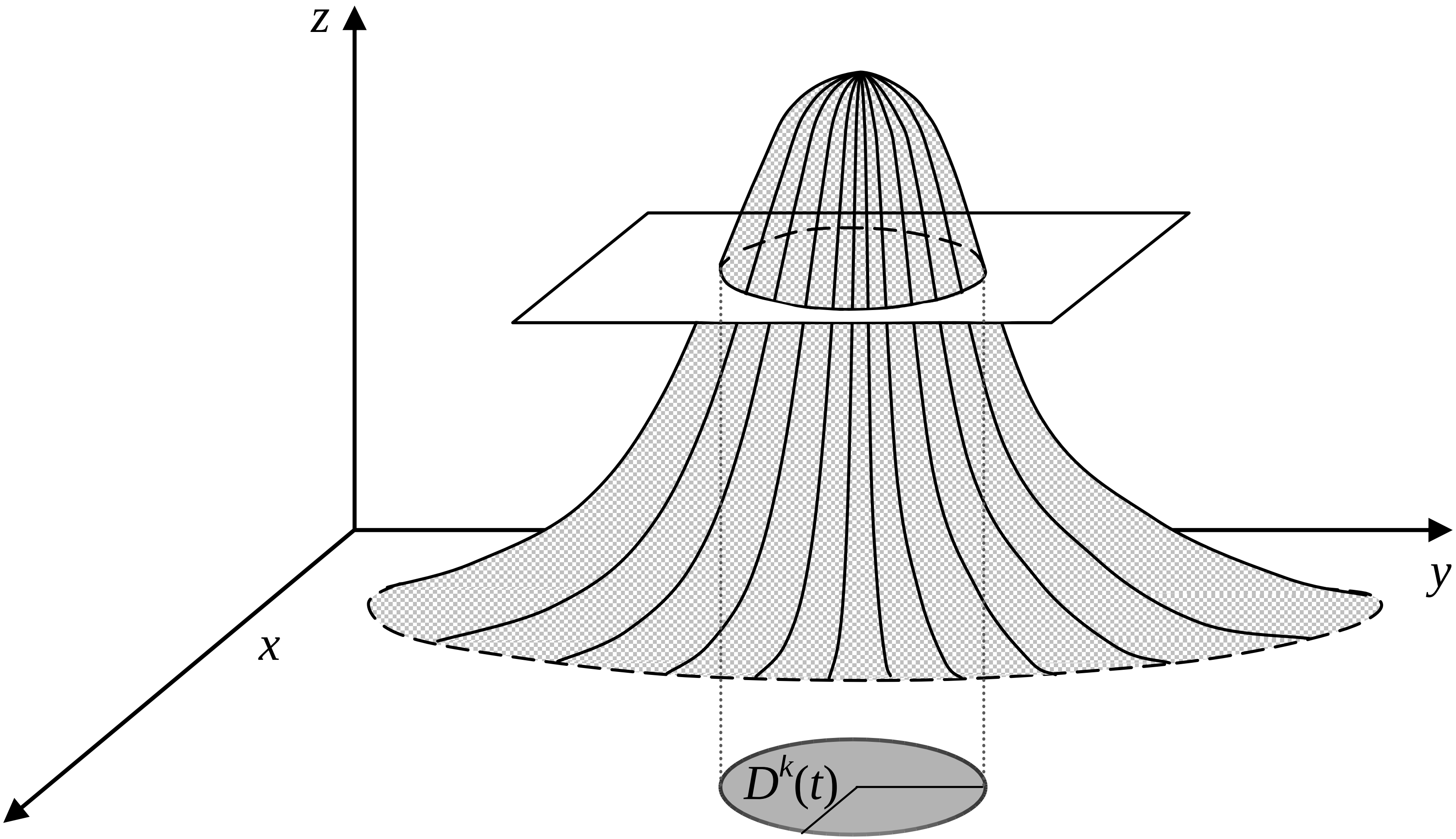

Figure. Domain $D^k(t)$ of integration bounded by an ellipse

4. MODELLING OIL SPILL DOMAIN

4.1 PROBABILISTIC APPROACH

The graph of the domain $D^k(t)$ is given in Figure below.

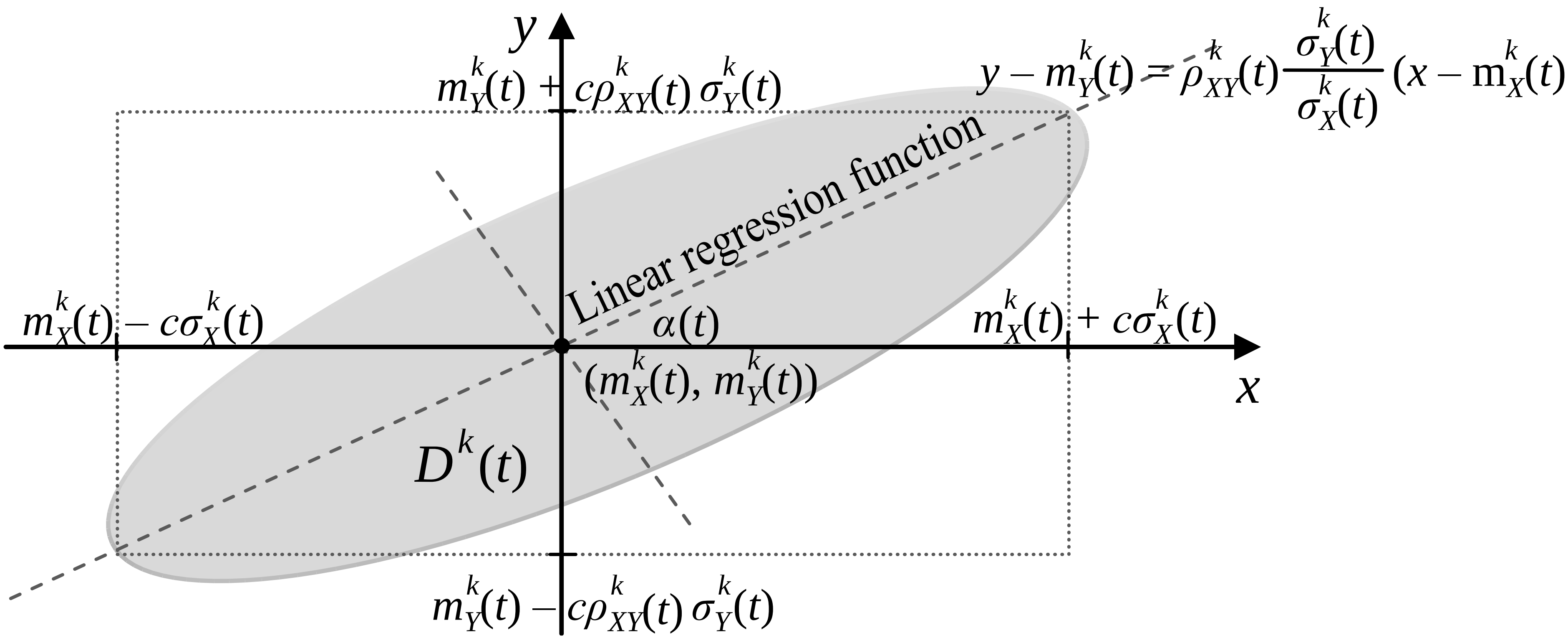

Figure. Domain $D^k(t)$ covering oil spill central point with probability p

4. MODELLING OIL SPILL DOMAIN
4.1 PROBABILISTIC APPROACH

Since

$$P((X^k(t),Y^k(t))\in D^k(t)) = 1-\exp[-\frac{1}{2}c^2],\ t \in \langle 0,T\rangle,$$

$k = 1,2,\ldots,m,$

then for a fixed probability *p*, the equality

$$p = P((X^k(t),Y^k(t))\in D^k),$$

holds if

$$c^2 = -2\ln(1-p).$$

4. MODELLING OIL SPILL DOMAIN
4.1 PROBABILISTIC APPROACH

Thus, the domain in which at the moment t the central point of oil spill is placed with the fixed probability p is given by

$$D^k(t) = \{(x,y) : \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2}$$

$$-\ 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}]$$

$$\leq -2\ln(1-p)\},\ t \in <0,T>,\ k = 1,2,\ldots,m.$$

4. MODELLING OIL SPILL DOMAIN
4.1 PROBABILISTIC APPROACH

Considering the above and the assumed in Section 3 definition of the central point of oil spill, for each fixed state k, $k = 1,2,\ldots,m$, of the process $A(t)$ and time $t \in <0,T>$, we define the oil spill domain

$$\bar{D}^k(t) = \{(x,y) : \frac{1}{1-(\rho_{XY}^k(t))^2}\left[\frac{(x-m_X^k(t))^2}{(\bar{\sigma}_X^k(t))^2} - 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\bar{\sigma}_X^k(t)\bar{\sigma}_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\bar{\sigma}_Y^k(t))^2}\right] \leq -2\ln(1-p)\},$$

where $\bar{\sigma}_X^k(t) = \sigma_X^k(t) + r^k(t)$, $\bar{\sigma}_Y^k(t) = \sigma_Y^k(t) + r^k(t)$,
and $r^k(t)$, is the radius of the oil spill domain $\bar{D}^k(t)$.

4. MODELLING OIL SPILL DOMAIN

4.1 PROBABILISTIC APPROACH

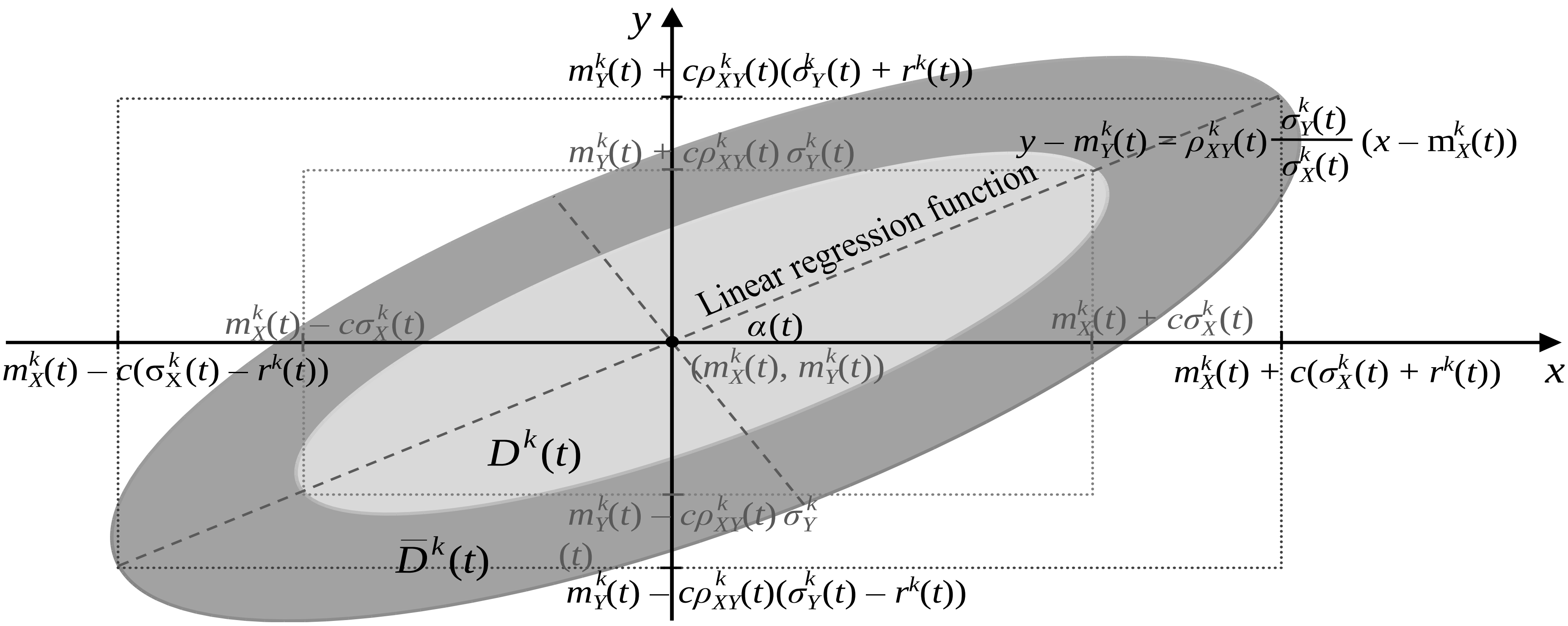

Figure. Oil spill domain $\overline{D}^k(t)$

4. MODELLING OIL SPILL DOMAIN

4.2 OIL SPILL DOMAIN FOR FIXED HYDRO-METEOROLOGICAL CONDITIONS

We suppose that the process $A(t)$ for all $t \in$ <0,T>, is at the fixed state k, $k = 1,2,\ldots,m$. Assuming a time step Δt and a number of steps s, $s \geq 1$, such that

$$(s-1)\Delta t < M_k \leq s\Delta t, \ \ s\Delta t \leq T,$$

where

$$M_k = E[\theta_k], \ k = 1,2...,m,$$

are the expected value of the process $A(t)$ sojourn times θ_k at the state k determined in Section 2, after multiple applying sequentially the procedure from Section 4.1, for

$$t = 1\Delta t, 2\Delta t, \cdots, s\Delta t,$$

4. MODELLING OIL SPILL DOMAIN
4.2 OIL SPILL DOMAIN FOR FIXED HYDRO-METEOROLOGICAL CONDITIONS

we receive the following sequence of oil spill domains

$$\overline{D}^k(\Delta t), \overline{D}^k(2\Delta t), \ldots, \overline{D}^k(s\Delta t).$$

Hence, the oil spill domain $\overline{D}^k$, $k = 1,2,\ldots,m,$ is described by the sum of determined domains of the sequence (4.14)

$$\overline{D}^k = \bigcup_{i=1}^{s} \overline{D}^k(i\Delta t) = \overline{D}^k(1\Delta t) \cup \overline{D}^k(2\Delta t) \cup \ldots \cup \overline{D}^k(s\Delta t),$$

and illustrated in the next slide.

4. MODELLING OIL SPILL DOMAIN

4.1 PROBABILISTIC APPROACH

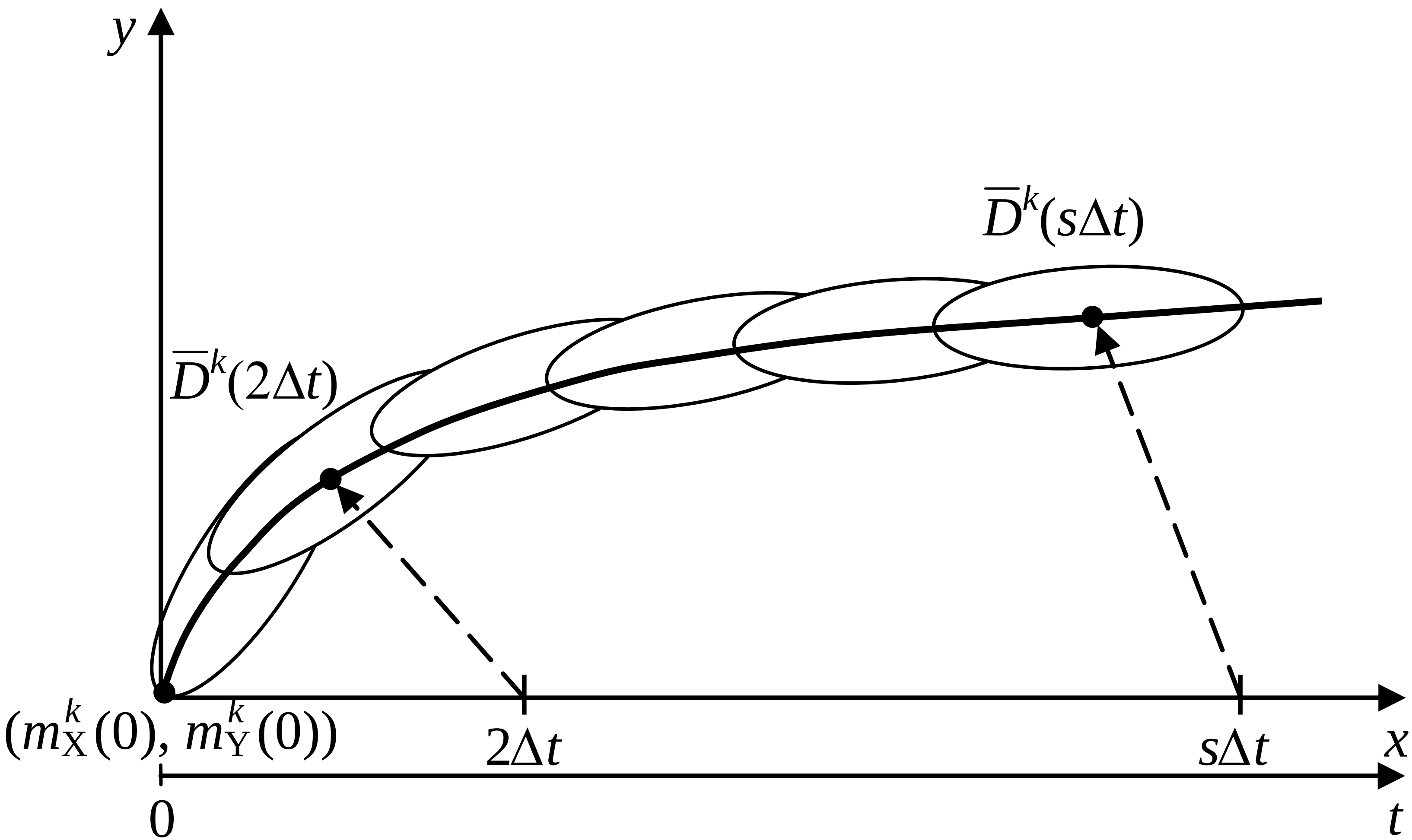

Figure. Oil spill domain for fixed hydro-meteorological conditions

4. MODELLING OIL SPILL DOMAIN
4.3 OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

We assume that the process of changing hydro-meteorological conditions in succession takes the states $k_1, k_2, \ldots, k_n$, $k_i \in \{1,2,\ldots,m\}$, $i = 1,2,\ldots,n$.

For a fixed time step Δt, after multiple applying sequentially the procedure from Section 4.1:

– for $t = 1\Delta t, 2\Delta t, \cdots, s_1\Delta t$, at the process $A(t)$ state k_1;

– for $t = (s_1 + 1)\Delta t, (s_1 + 2)\Delta t, \cdots, s_2\Delta t$, at the process $A(t)$ state k_2;

...

– for $t = (s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t, \cdots, s_n\Delta t$, at the process $A(t)$ state k_n;

4. MODELLING OIL SPILL DOMAIN
4.3 OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

we receive the following sequence of oil spill domains:

$$\overline{D}^{k_1}(1\Delta t),\ \overline{D}^{k_1}(2\Delta t), \ldots, \overline{D}^{k_1}(s_1\Delta t),$$

$$\overline{D}^{k_2}((s_1+1)\Delta t),\ \overline{D}^{k_2}((s_1+2)\Delta t), \ldots, \overline{D}^{k_2}(s_2\Delta t),$$

…

$$\overline{D}^{k_n}((s_{n-1}+1)\Delta t),\ \overline{D}^{k_n}((s_{n-1}+2)\Delta t), \ldots, \overline{D}^{k_n}(s_n\Delta t),$$

4. MODELLING OIL SPILL DOMAIN

4.3 OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

in which, respectively at the moments:

$$1\Delta t, 2\Delta t, \cdots, s_1\Delta t,$$
$$(s_1 + 1)\Delta t, (s_1 + 2)\Delta t, \cdots, s_2\Delta t,$$
$$\ldots$$
$$(s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t, \cdots, s_n\Delta t,$$

where s_i, $i = 1,2,\ldots,n$, are such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{i} M_{k_j\, k_{j+1}} \leq s_i\Delta t, \; i = 1,2,\ldots,n, \; s_n\Delta t \leq T,$$

4. MODELLING OIL SPILL DOMAIN

4.3 OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

and

$M_{k_j\,k_{j+1}} = E[\theta_{k_j\,k_{j+1}}]$, $j = 1,2...,n$-1,

are the expected value of the process $A(t)$, $t \in <0,T>$, conditional sojourn times

$\theta_{k_j\,k_{j+1}}$, $j = 1,2...,n-1$

at the states k_j, upon the next state is k_{j+1}, $j = 1,2...,n$-1, k_j, k_{j+1}, $\in \{1,2,...,m\}$, $j = 1,2...,n$-1, determined in Section 2.

4. MODELLING OIL SPILL DOMAIN

4.3 OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

Hence, the oil spill domain $\overline{D}^{k_1,k_2,\ldots,k_n}$, $k_1, k_2, \ldots, k_n \in \{1,2,\ldots,m\}$, is described by the sum of determined domains of the sequences, given by

$$\overline{D}^{k_1,k_2,\ldots,k_n} = \bigcup_{i=1}^{n}\bigcup_{j=1}^{s_i}\overline{D}^{k_i}\left((s_{i-1}+j)\Delta t\right) = [\overline{D}^{k_1}(1\Delta t)\cup\overline{D}^{k_1}(2\Delta t)\cup\ldots\cup\overline{D}^{k_1}(s_1\Delta t)]$$

$$+\ [\overline{D}^{k_2}\left((s_1+1)\Delta t\right)\cup\overline{D}^{k_2}\left((s_1+2)\Delta t\right)\cup\ldots\cup\overline{D}^{k_2}(s_2\Delta t)]$$

...

$$+\ [\overline{D}^{k_n}\left((s_{n-1}+1)\Delta t\right)\cup\overline{D}^{k_n}\left((s_{n-1}+2)\Delta t\right)\cup\ldots\cup\overline{D}^{k_n}(s_n\Delta t)]$$

$k_1, k_2, \ldots, k_n \in \{1,2,\ldots,m\}$,

4. MODELLING OIL SPILL DOMAIN
4.1 PROBABILISTIC APPROACH

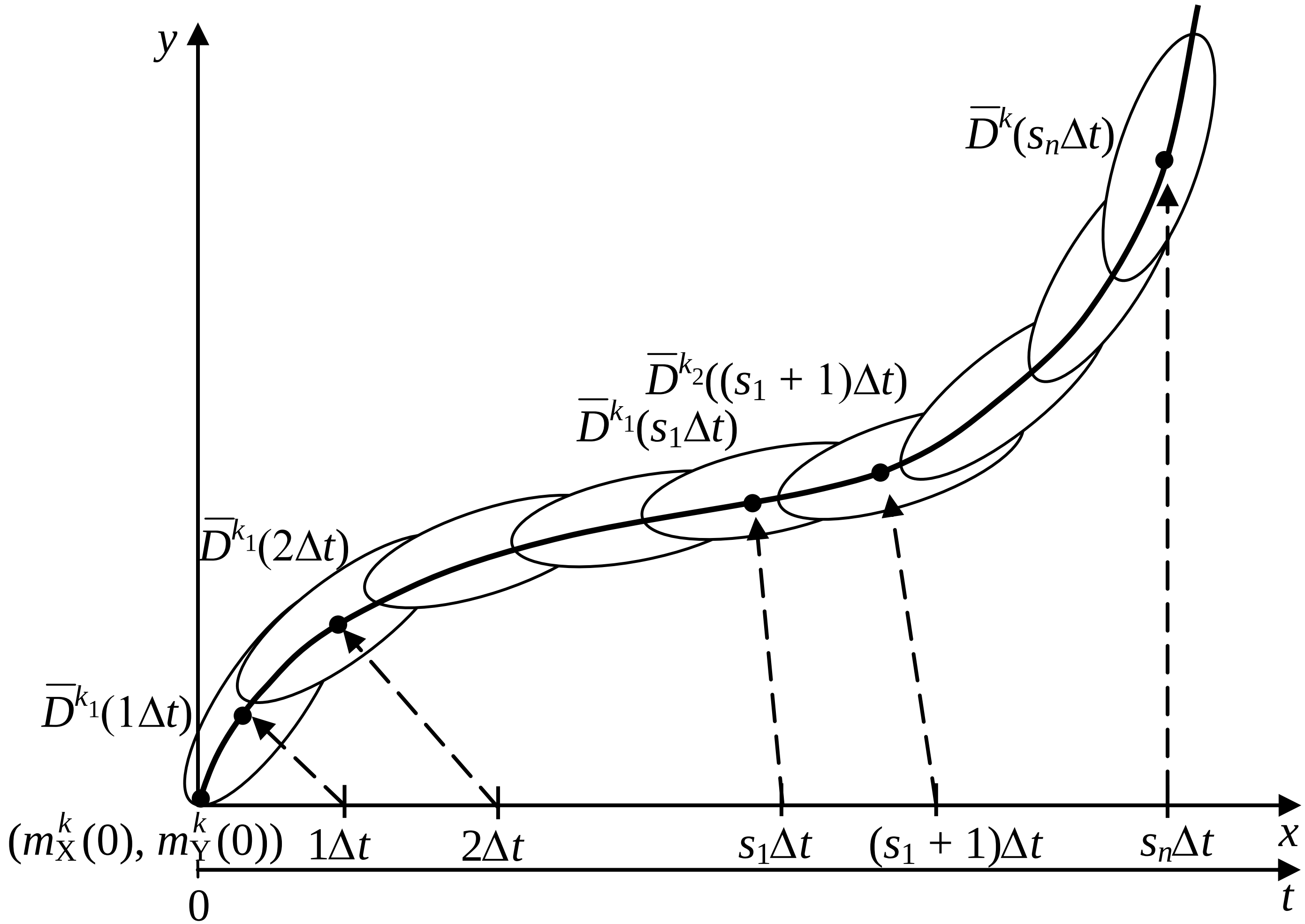

Figure. Oil spill domain for changing hydro-meteorological conditions

5. MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

The general model of the process of changing hydro-meteorological conditions at oil spill area is proposed in Section 2 and defined by the initial probabilities at its states, the probabilities of transitions between these states and the distributions of the conditional sojourn times at these states.

Moreover, its main characteristics, i.e. the mean values and variances of the unconditional sojourn times, the limit values of transient probabilities and the unconditional mean values of total sojourn times at the particular states T for the fixed time are determined.

In this section, a Monte Carlo simulation approach is applied to the process of changing hydro-meteorological conditions at oil spill area prediction and its characteristics evaluation.

5. MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

5.1 GENERATING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

Details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.

5. MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

5.2 GENERAL PROCEDURE OF MONTE CARLO SIMULATION APPLICATION TO CHARACTERISTICS DETERMINATION OF A PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

Details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.

5. MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

5.3. OIL SPILL DOMAIN IN VARYING HYDRO-METEROLOGICAL CONDITIONS MONTE CARLO SIMULATION PREDICTION

Using the procedures of the process of changing hydro-meteorological conditions at oil spill area prediction described in Sections 5.1-5.2 and the modified method of the domain of oil spill determination presented in Section 4.3 the Monte Carlo simulation oil spill domain prediction can be done.

The modified method of the domain of oil spill determination presented in Section 4.3 depends on changing the procedure by replacing its conditions by the modified conditions:

5. MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN
5.3. OIL SPILL DOMAIN IN VARYING HYDRO-METEROLOGICAL CONDITIONS MONTE CARLO SIMULATION PREDICTION

The symbols s_i, $i = 1,2,...,n$, are such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{i} t_{k_j\, k_{j+1}} = s_i \Delta t,\ i = 1,2,...,n,\ s_n \Delta t \le T,$$

and

$$t_{k_j\, k_{j+1}},\ j = 1,2...,n\text{-}1,$$

are the realizations of the process $A(t)$, $t \in \langle 0,T\rangle$, conditional sojourn times $\theta_{k_j\, k_{j+1}}$, $j = 1,2...,n-1$ at the states k_j, upon the next state is k_{j+1}, $j = 1,2...,n-1$, k_j, k_{j+1}, $\in \{1,2,...,m\}$, $j = 1,2...,n-1$, defined in Section 2.

6. IDENTIFICATION OF PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS

Details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.

7. IDENTIFICATION OF OIL SPILL DOMAIN DRIFT TREND

Details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.

8. CONCLUSIONS

Details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.

REFERENCES

Details are given in Journal of Polish Safety and Reliability Association – JPSRA, Special Issue on HAZARD Project, Volume 10, No 1, April 2019.