Optimal Load-distribution for Nonplanar Wings

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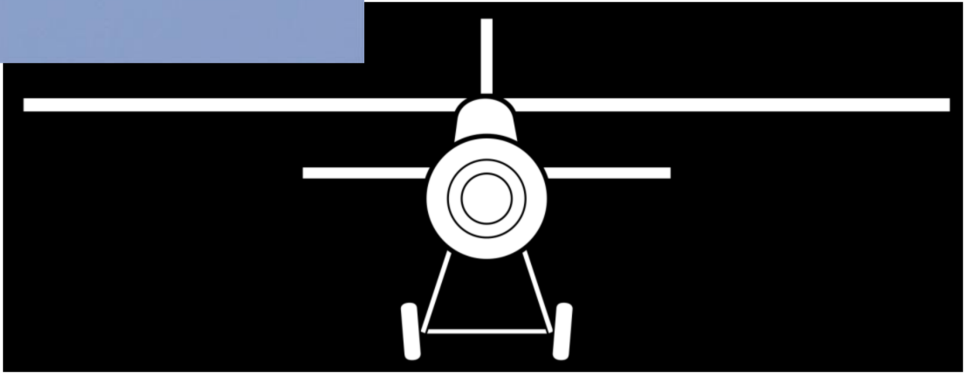
The Problem Statement

winglet “hype”, Ingo Rechenberg, Kurt,..

Nature tends to be efficient

Is a non-planar wing “better”



The Problem Statement II

- There is much misunderstanding of how wing tips, winglets, feathered wing tips etc. do work
- This sometimes leads to funny concepts (which obviously do not work)
- A “wing tip vortex” is a misleading view (the wing trailing edge vortex sheet roles up itself and only looks like that after a while)

- -> requires a close view into vortex dynamics

What is Efficiently Flying ?

Flying heavier than air (winged aircraft, helicopter, ..) is an "application" of Newton's second law:

Throwing a certain amount of air mass every second (dm/dt) with a velocity vs downwards yields a lifting force L

$$L=dm/dt*vs$$

Requires constant power P, however: $P=dEkin/dt= ½*dm/dt*vs^2$

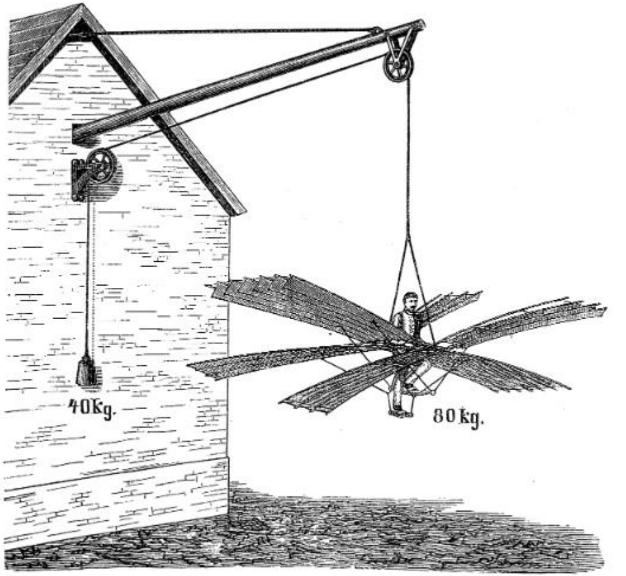
So efficient flying means **low vs** and **large dm/dt → large area** !

Required power shows up in aircraft as **induced drag D** $P=D*v$

(Now very inefficient VTOL concepts around, especially with electric drives with accumulators ! =combining the worst with the worst in flying, good for wasting energy)

The most efficient VTOL's ever flown

Otto Lilienthal (1864), Aerovelo (2013)



Lift/Drag

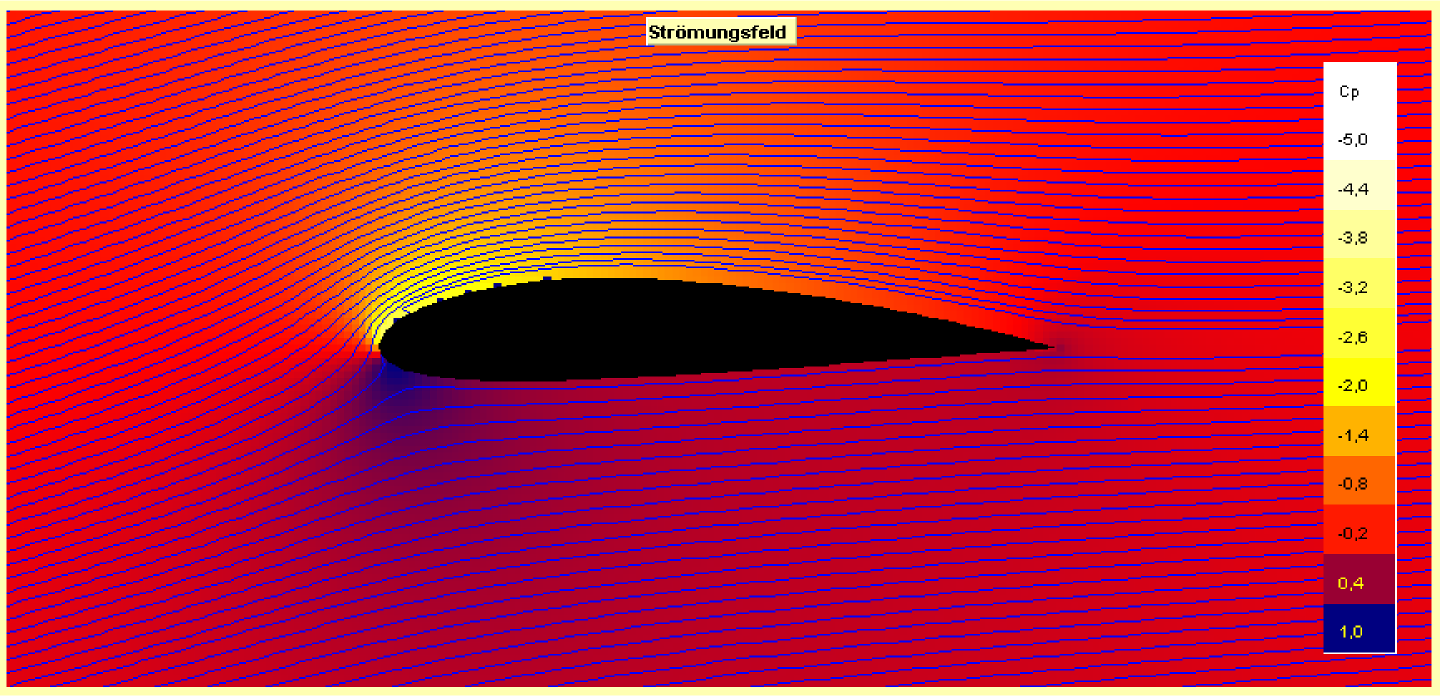
"Better": lower drag for a required lift for a given span, weight

An aircraft is no bluff body, so no pressure drag, so

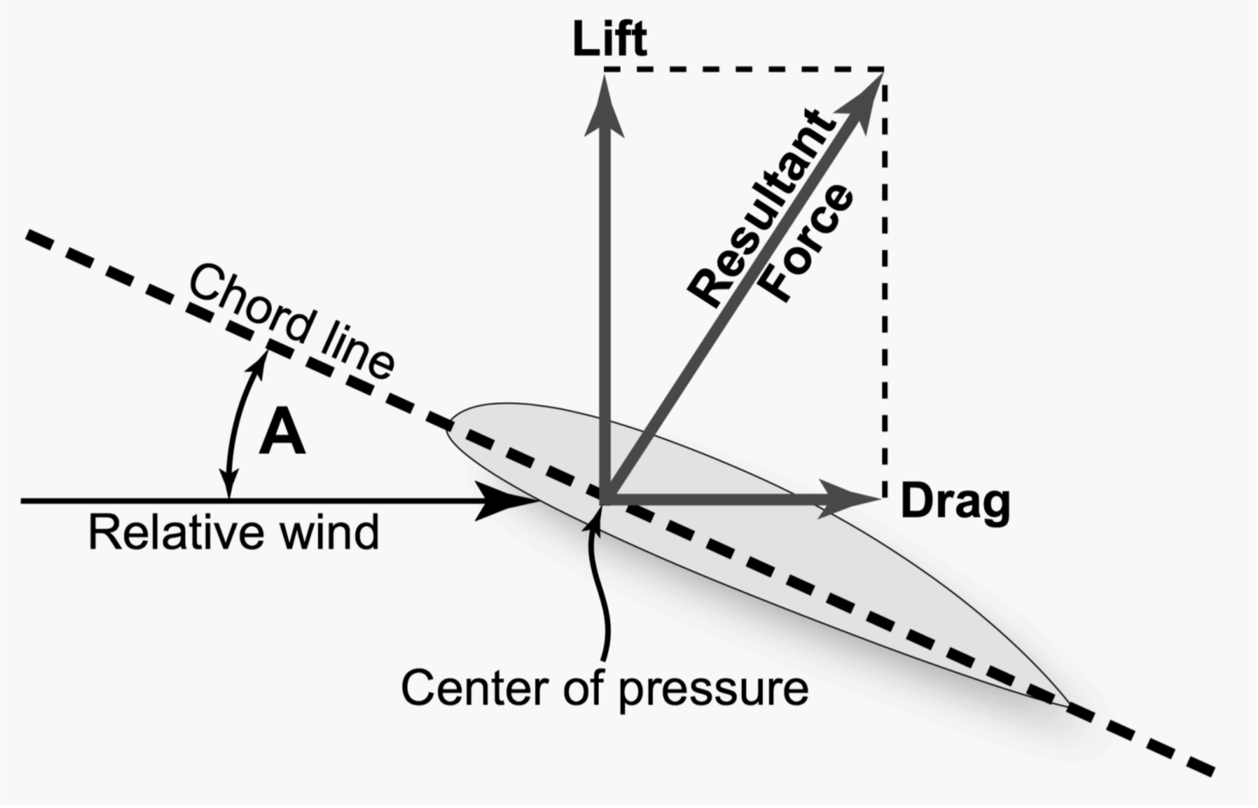
drag = viscous drag in boundary layer + induced drag

Understanding "induced drag" is the key for our problem solution

The Infinite Wing



Lift/Drag

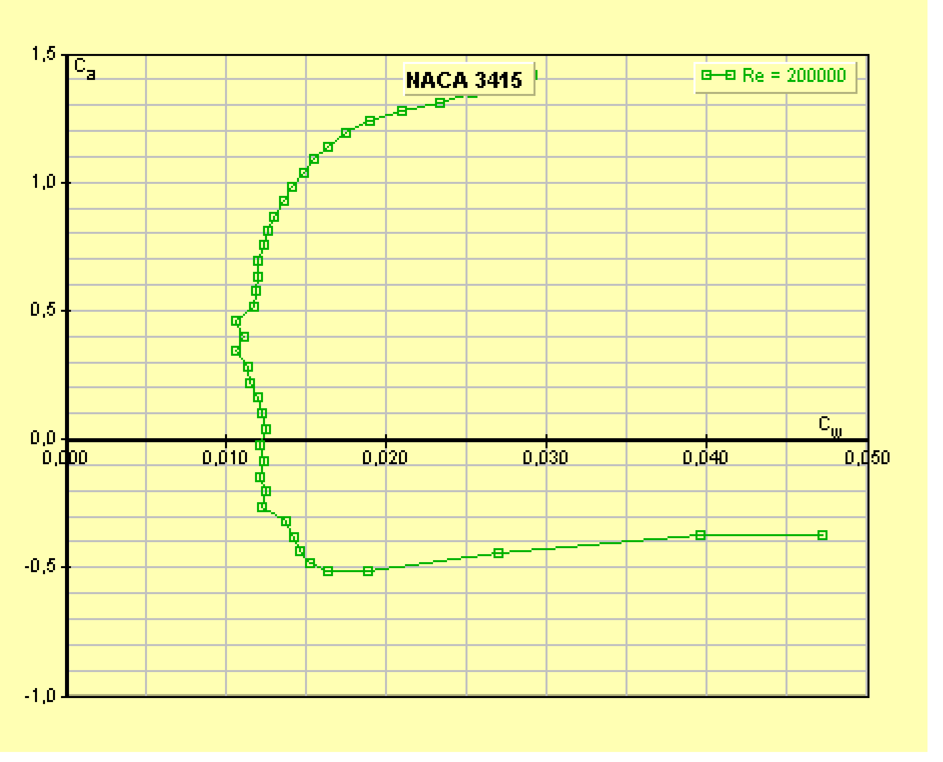


Lift
Resultant Force
Chord line
A
Drag
Relative wind
Center of pressure

Lift and Drag of an Airfoil

Lift=1/2*ca*A*v^2

Dvisc=1/2*cw*A*v^2



Observation

Power is also required for the kinetic energy in vortices → keep velocity in vortices low for low power consumption

Induced Drag Formulas

Cind=ca^2/(k*π*AR)   Dind=1/2*cind*A*v^2

K: factor depending for circulation/lift distribution and shape of wing seen from behind (non-planar); k=1 for elliptical distribution and planar wing;

Goal is to get k-factor >1 by non-planar wings

AR=A/b^2 :   aspect ratio (b: wing span), span/chord for rectangular wing

Lift/drag= ca/(cvisc+cind)

L/D largest, if cind=cvisc   (best flight range), so cind is important !

The Views on Flow

Three views on the same flow situation:

- Pressure distribution
- Velocity distribution
- Vortex distribution

All connected:

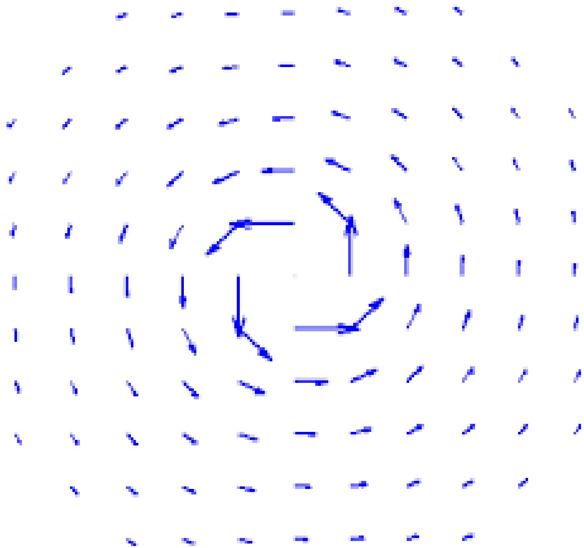
Velocity - pressure (Bernoulli's law $pt = p + \frac{1}{2} * v^2$)

Vortex - velocity (Biot-Savart law)

Vortex I

Potential vortex

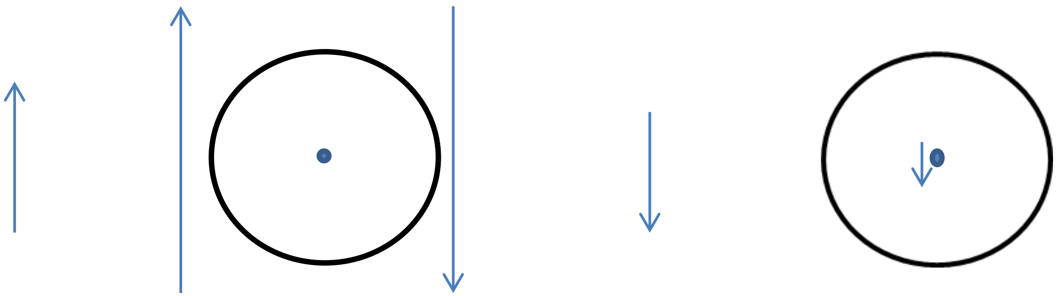
v ~ 1/r



Air flows in circles around a (thought) axis -> vortex filament

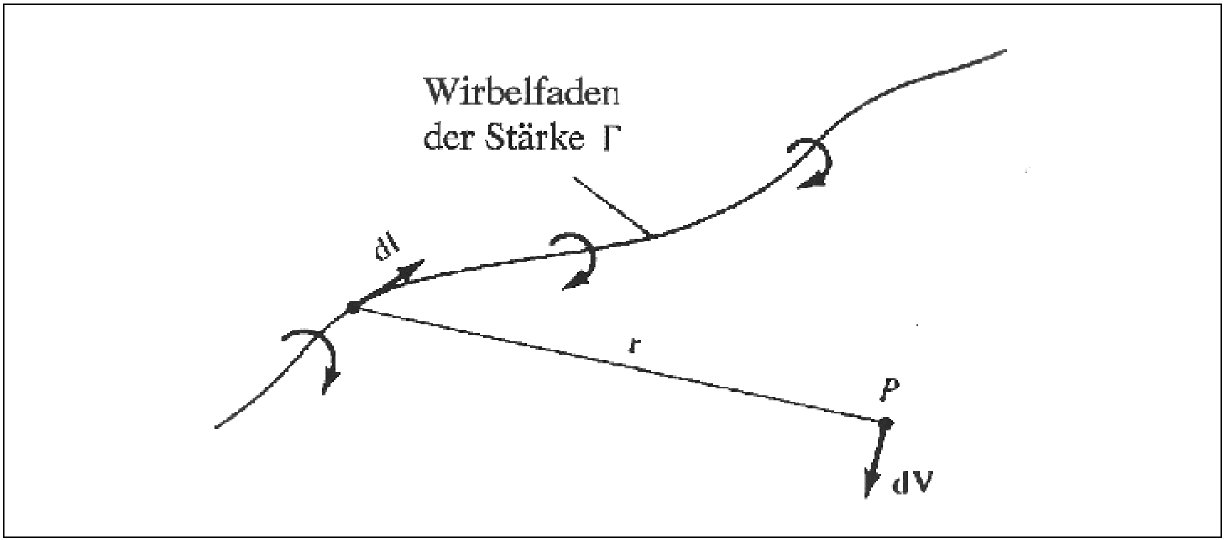
Vortex Ring Visualized by Smoke

Why is a Vortex Ring Moving through Space ?



Vortex II

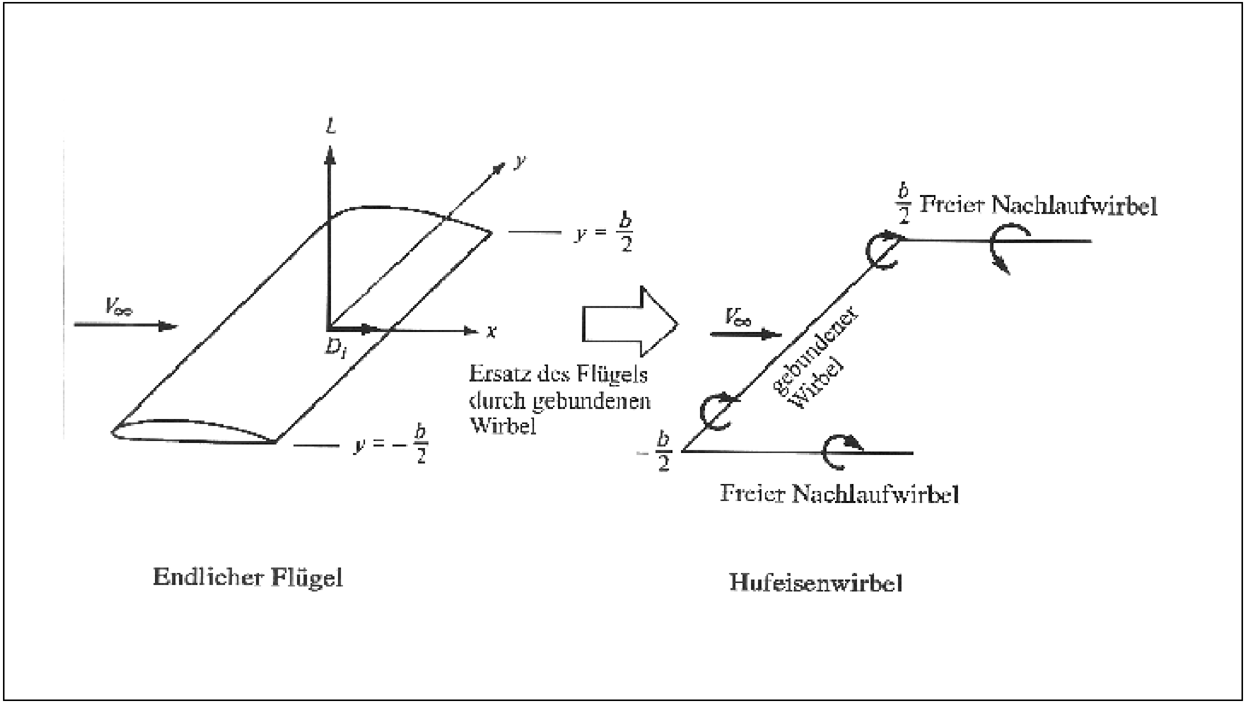
- Vortex filaments are always closed or end at walls (move with air velocity)
- The strength of vortex filaments stays constant (in nonviscous flow)



vortex filament = electric current velocity = magnetic field

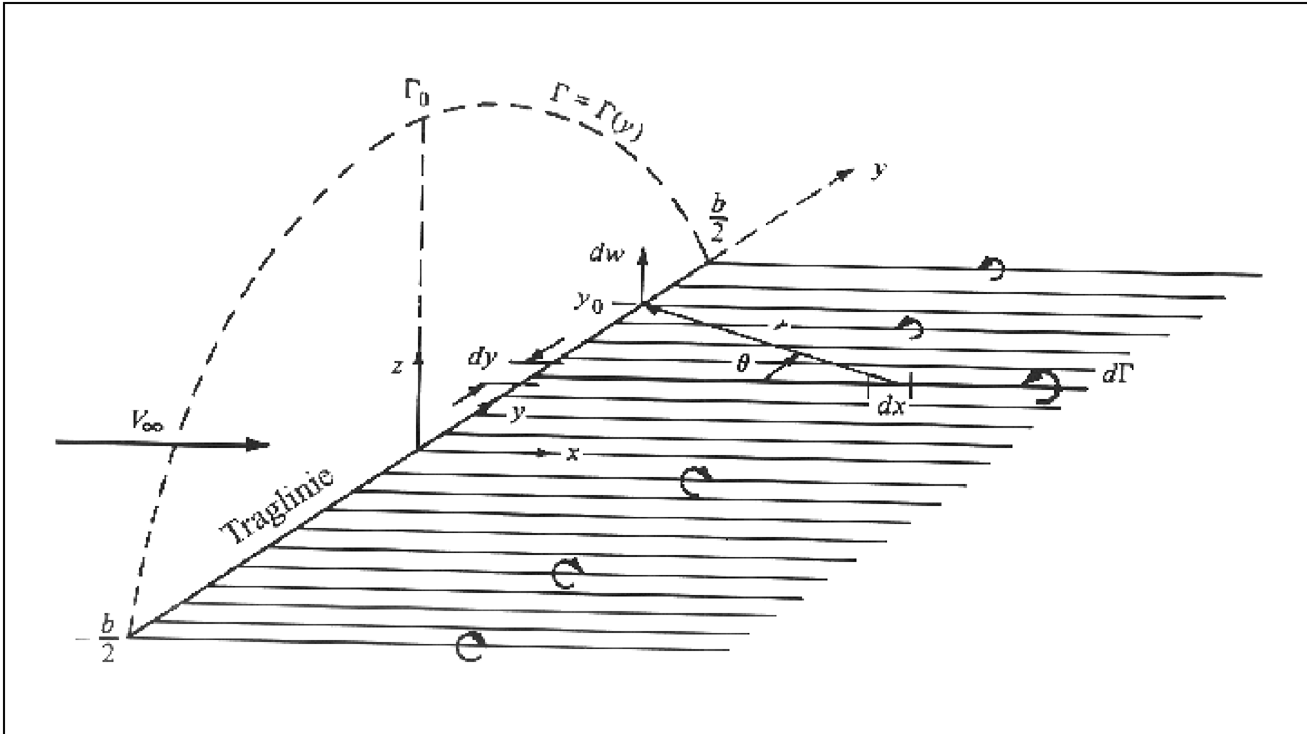
(Biot-Savart law)

Vortex III



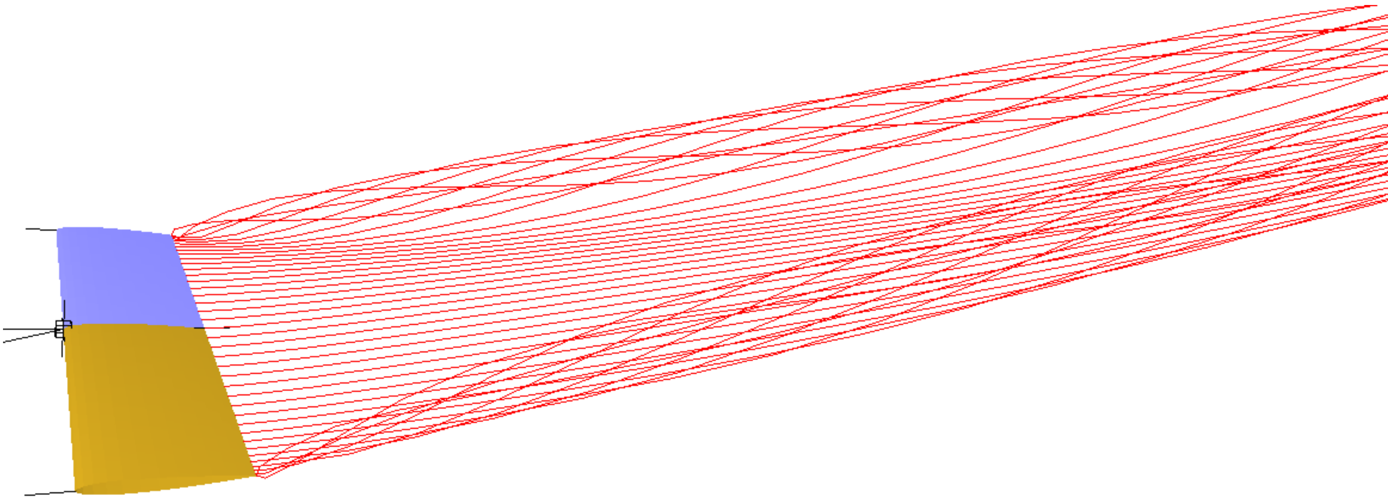
Horse-shoe vortex closed by starting vortex very far away

Vortex Sheet



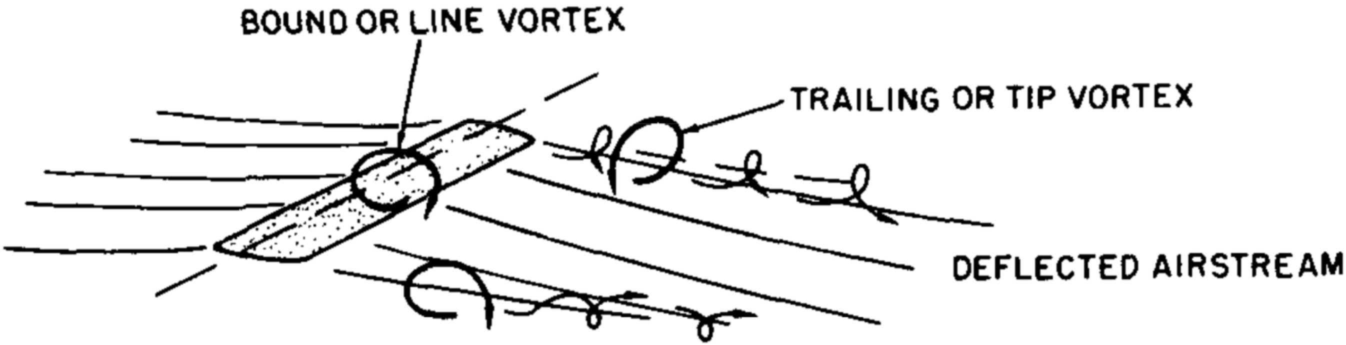
approximately moves with inflow, has shape of wing

Vortex Sheet II



Program demonstration !

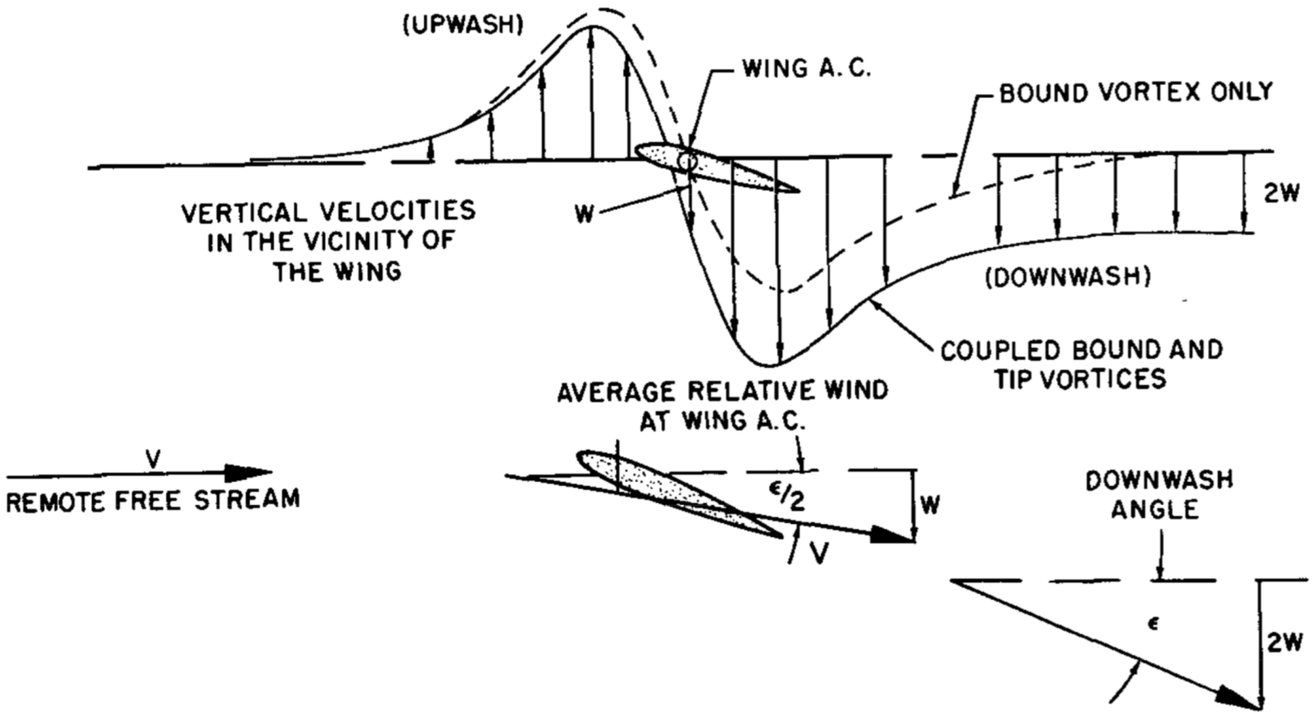
Vortex Sheet II



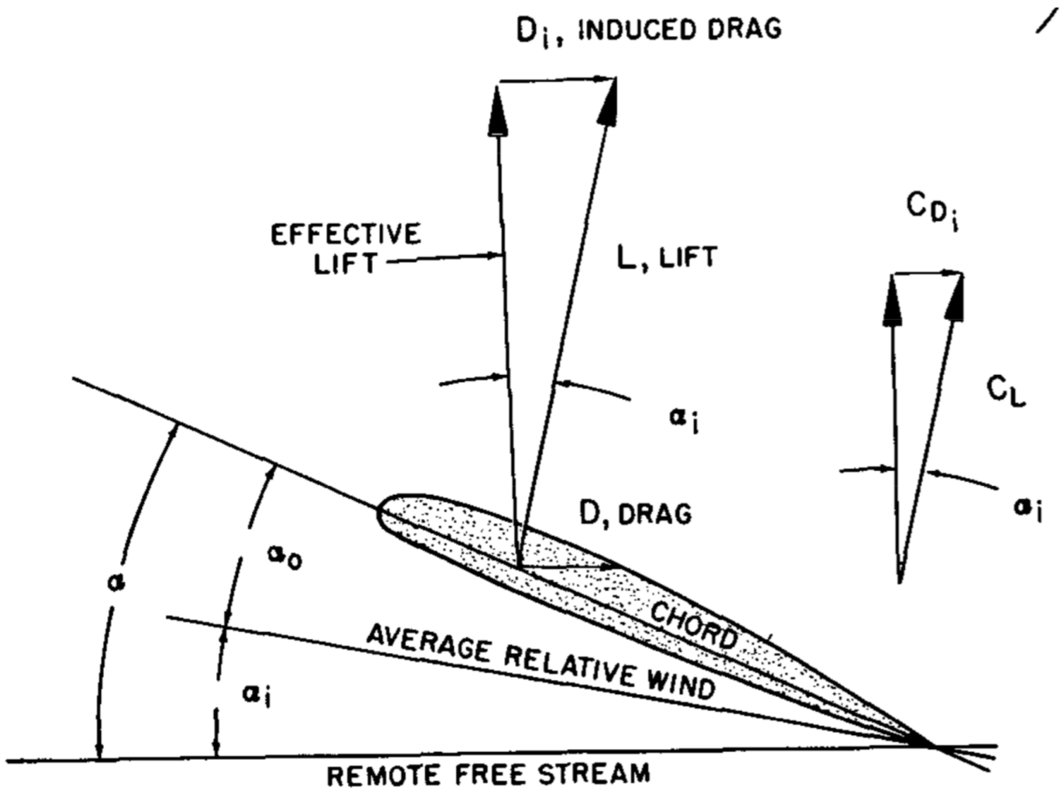
Γ orthogonal to incoming free flow -> lift   sectional lift ~ Γ*v
Γ parallel to incoming free flow -> downwash velocity

-> the vortex sheet generates a downwash superimposed on the free+bound vortex flow

Downwash -> Induced Angle -> Induced Drag



Downwash -> Induced Drag



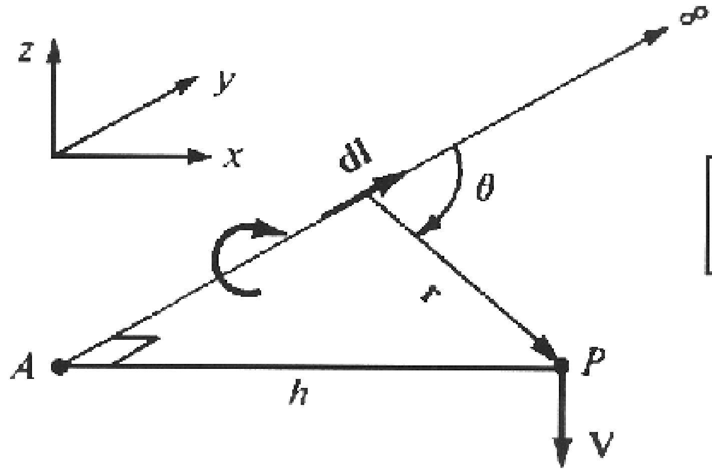
Lowest Total Induced Drag

Munk’s stagger theorem

Total drag lowest, if lowest kinetic energy for lift → constant downwash velocity distribution along wing span (Munk’s second theorem) (→ elliptic circulation distribution for a planar wing)

→ Have to investigate the vortex sheet strength of the non-planar vortex sheet of a non-planar wing

Velocity distribution of half infinitesimal vortex filament section

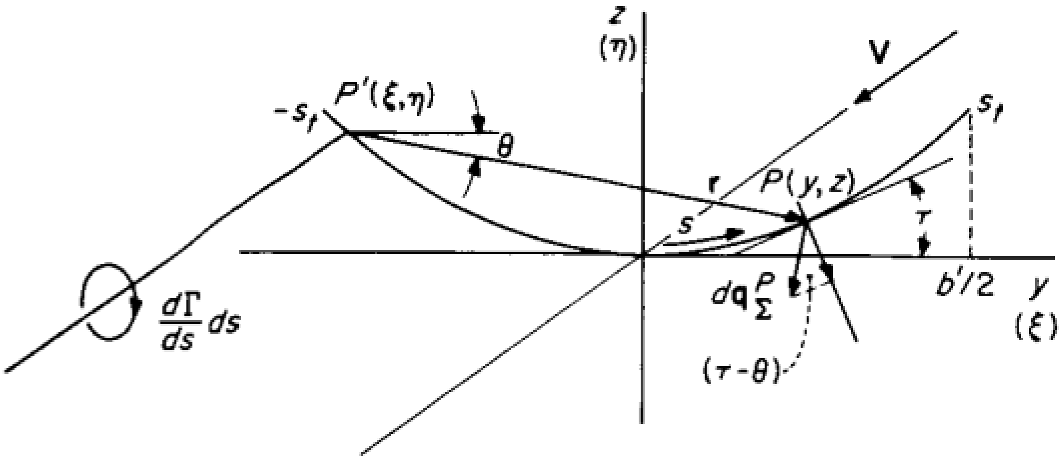


$$V = \frac{\Gamma}{4\pi h}$$

Velocity contribution to flow field at wing location orthogonal to free flow direction ! Always vertical for a planar wing (worst situation)

Better: distribute vortex filament sections over space (non-planar) -> non-vertical contributions -> lower local induced drag -> lower total induced drag

Downwash for Nonplanar Wing



Munk: best, if component normal to local wing section is w0*cos(tau), w0 being constant over wing span, then minimum total induced drag

Now a lot of math is omitted … see:

Clarence D. Cone, NASA Technical Report R-139, 1962

Span of a Non-planar Wing

We understand the span of a non-planar wing geometry as the span of the projection on a horizontal plane, the area of a non-planar wing is it’s projected area

-> wing length is longer than span in general

-> more viscous drag in general for a change in shape because of larger wing area (has to be traded against lower induced drag just in case)

K-factor and it's Electric Analogy

Then, the k-factor is:

$$K = \frac{\left(\frac{\Gamma_o}{w_o}\right)}{\left(\frac{b'}{2}\right)} \int_{-1}^{1} \frac{\Gamma}{\Gamma_o} \, d\gamma$$

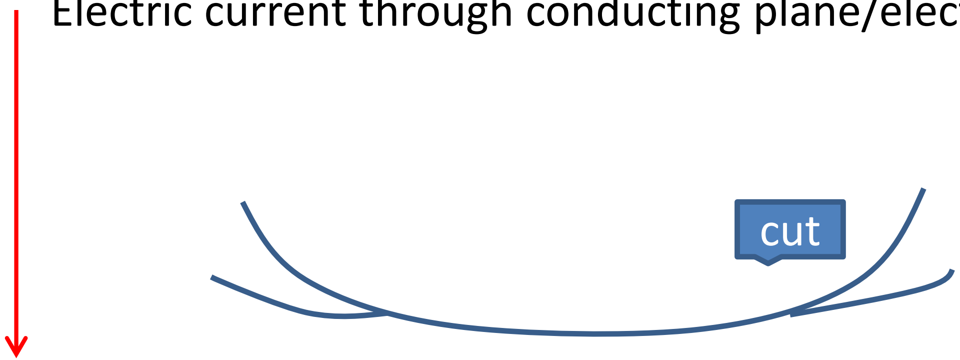
Because of Laplace-equation, analogy to electric potential E (voltage):

$$K_e = \frac{(\Delta E)_o}{\left(\frac{dE}{dz}\right)_\infty \frac{b'}{2}} \int_{-1}^{1} \frac{\Delta E}{(\Delta E)_o} \, d\gamma$$

Electrically Conducting Plane/Electrolytic trough

Voltage +E

Electric current through conducting plane/electrolytic trough



Understanding the Formula

$$K_e = \frac{(\Delta E)_o}{\left(\frac{dE}{dz}\right)_\infty \frac{b'}{2}} \int_{-1}^{1} \frac{\Delta E}{(\Delta E)_o} \, d\gamma$$

Delta E0: Voltage jump in middle of “wing shape” cut

Delta E: Voltage jump along “wing shape” cut

dE/dz: E-field=Voltage between lines divided by distance

Replace integral by summing over voltage differences along the “wing shape” cut

More intuitive view: Replace electric current by water flow, the “wing shape” cut by an appropriate vertical wall stopping the water and delta E by water level difference across wall

Consequences and advantages

- All non-planar shapes have a lower induced drag than the planar one for a given span

- The improvement in induced drag of a given wing shape is easily estimated

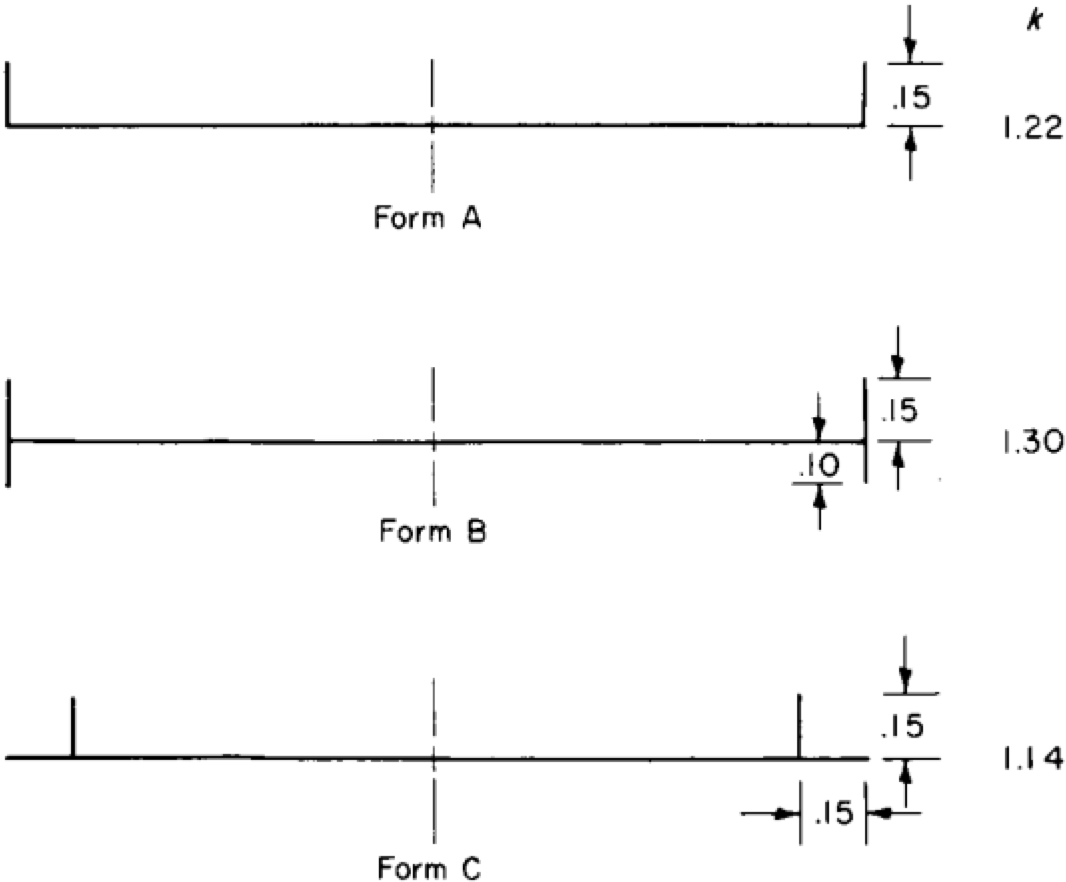
- The optimal circulation distribution for lowest induced drag is automatically given → optimal wing designs by using vortex lattice methods now easy

Example: Feathered wing tips of “birds” now possible to design optimally

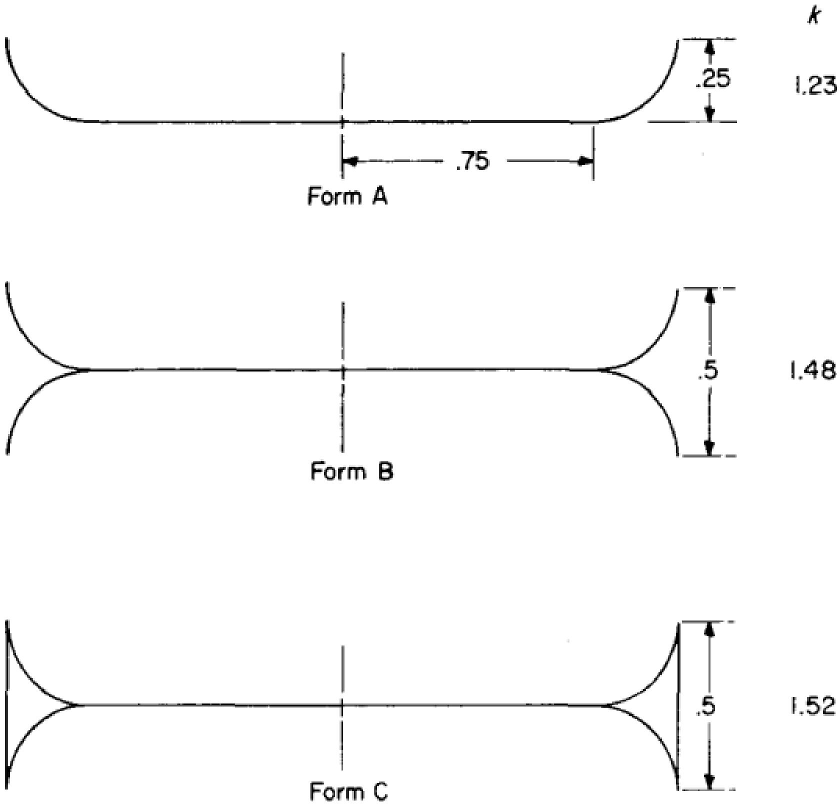
- Because only a 2-dimensional Laplace equation has to be solved, this can be done numerically in a short program

- The non-mathematics may use a conducting liquid (salt in water), a power supply, a voltmeter, several wires and stripes of isolating material, which could be bent (PVC, for instance) to the shape of the non-planar “wing”

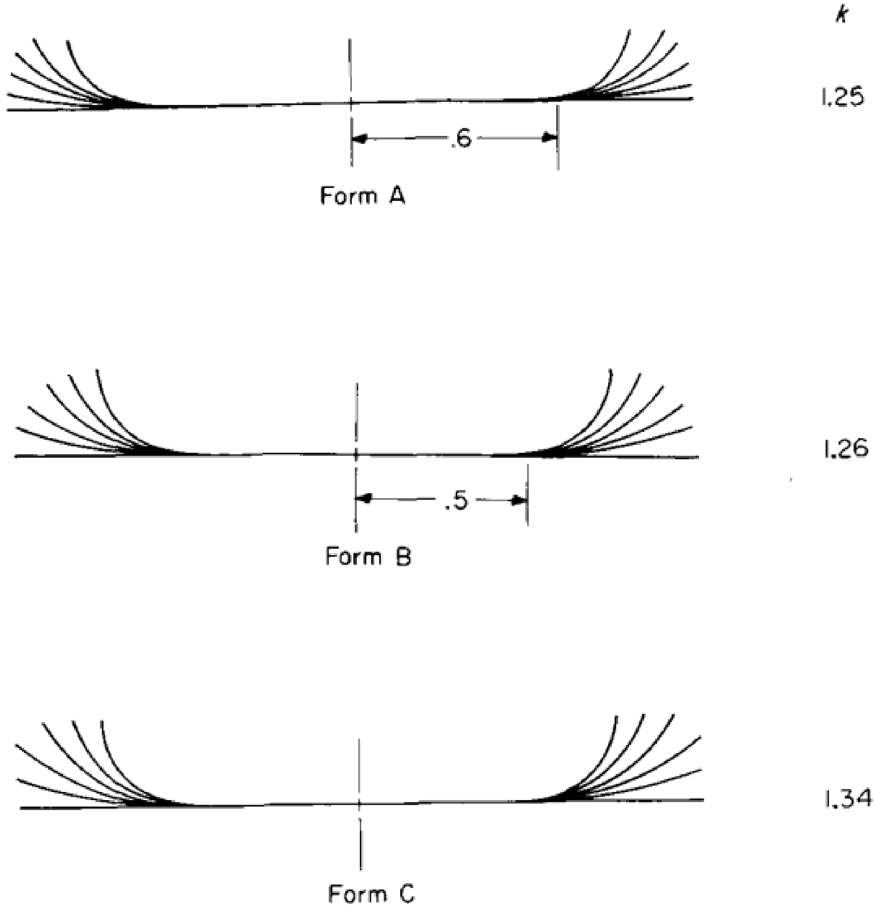
Examples I



Examples II



Examples III



Results

- Non-planar wings always have a lower induced drag than planar ones
- Optimal circulation distribution easily calculated/measured
- Cave ! Viscous drag may go up due to larger real wing area; birds therefor only use their feathered wing tips at large ca and attack angle, otherwise “winglets” are flown “wing-integrated”

- Feathered wing tips, winglets and such, how do they work ? How to design optimally ? A problem, which occurred quite a few times in the last decades, has already been solved about 70 years ago, specific problems solved by using an “analog computer”: Clarence D. Cone, NASA Technical Report R-139, 1962

Thank you for your attention!